CHAPTER 7
Coordinate Geometry

1. Find the distance between the following pairs of points:
(i) (2, 3), (4,1)
(ii) (–5, 7), (–1, 3)
(iii) (a, b), (–a, –b)

Ans. (i) Applying Distance Formula to find distance between points (2, 3) and (4,1), we get

d = $\sqrt{(4-2)^2+(1-3)^2} = \sqrt{(2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$ units

(ii) Applying Distance Formula to find distance between points (–5, 7) and (–1, 3), we get

d = $\sqrt{[-1-(-5)]^2+(3-7)^2} = \sqrt{(4)^2+(-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$ units

(iii) Applying Distance Formula to find distance between points (a, b) and (–a, –b), we get

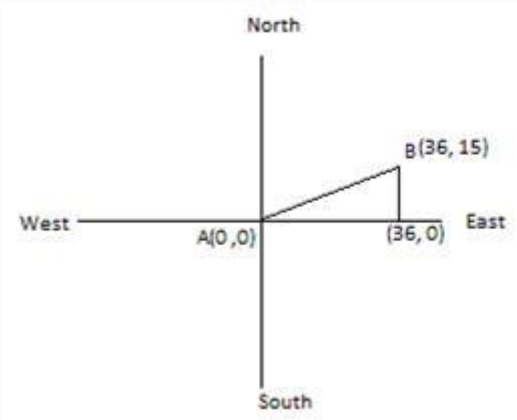
d = $\sqrt{(-a-a)^2+(-b-b)^2} = \sqrt{(-2a)^2+(-2b)^2} = \sqrt{4a^2+4b^2} = \sqrt{4(a^2+b^2)} = 2\sqrt{a^2+b^2}$

2. Find the distance between the points (0, 0) and (36, 15). Also, find the distance between towns A and B if town B is located at 36 km east and15 km north of town A.

Ans. Applying Distance Formula to find distance between points (0, 0) and (36, 15), we get

d = $\sqrt{(36-0)^2+(15-0)^2} = \sqrt{(36)^2+(15)^2} = \sqrt{1296+225} = \sqrt{1521} = 39$

Town B is located at 36 km east and15 km north of town A. So, the location of town A and B can be shown as:



Clearly, the coordinates of point A are (0, 0) and coordinates of point B are (36, 15).

To find the distance between them, we use Distance formula:

d = $\sqrt{[36-0]^2+(15-0)^2} = \sqrt{(36)^2+(15)^2} = \sqrt{1296+225} = \sqrt{1521} = 39$ km

3. Determine if the points (1, 5), (2, 3) and (–2, –11) are collinear.

Ans. Let A = (1, 5), B = (2, 3) and C = (–2, –11)

Using Distance Formula to find distance AB, BC and CA.

AB = $\sqrt{[2-1]^2+(3-5)^2} = \sqrt{(1)^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$

BC = $\sqrt{[-2-2]^2+(-11-3)^2} = \sqrt{(-4)^2+(-14)^2} = \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$

CA = $\sqrt{[-2-1]^2+(-11-5)^2} = \sqrt{(-3)^2+(-16)^2} = \sqrt{9+256} = \sqrt{265}$

Since AB + AC $\neq$ BC, BC + AC $\neq$ AB and AC $\neq$ BC.

Therefore, the points A, B and C are not collinear.

4. Check whether (5, –2), (6, 4) and (7, –2) are the vertices of an isosceles triangle.
Ans. Let A = (5, –2), B = (6, 4) and C = (7, –2)
Using Distance Formula to find distances AB, BC and CA.

AB = $\sqrt{[6-5]^2+[4-(-2)]^2} = \sqrt{(1)^2+(6)^2} = \sqrt{1+36} = \sqrt{37}$

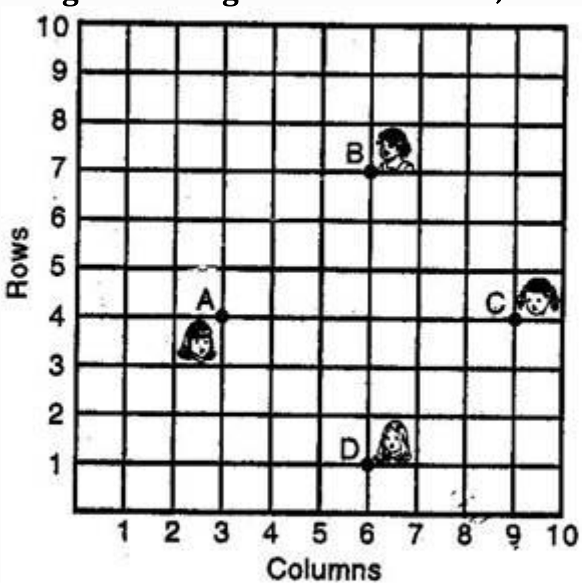
BC = $\sqrt{[7-6]^2+(-2-4)^2} = \sqrt{(1)^2+(-6)^2} = \sqrt{1+36} = \sqrt{37}$

CA = $\sqrt{[7-5]^2+[-2-(-2)]^2} = \sqrt{(2)^2+(0)^2} = \sqrt{4+0} = \sqrt{4} = 2$

Since AB = BC.

Therefore, A, B and C are vertices of an isosceles triangle.

5. In a classroom, 4 friends are seated at the points A (3, 4), B (6, 7), C (9, 4) and D (6, 1). Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli. "Don't you think ABCD is a square?"Chameli disagrees. Using distance formula, find which of them is correct.



Ans. We have A = (3, 4), B = (6, 7), C = (9, 4) and D = (6, 1)
Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[6-3]^2+[7-4]^2} = \sqrt{(3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

BC = $\sqrt{[9-6]^2+[4-7]^2} = \sqrt{(3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

CD = $\sqrt{[6-9]^2+[1-4]^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

DA = $\sqrt{[6-3]^2+[1-4]^2} = \sqrt{(3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Therefore, All the sides of ABCD are equal here. … (1)

Now, we will check the length of its diagonals.

AC = $\sqrt{[9-3]^2+[4-4]^2} = \sqrt{(6)^2+(0)^2} = \sqrt{36+0} = 6$

BD = $\sqrt{[6-6]^2+[1-7]^2} = \sqrt{(0)^2+(-6)^2} = \sqrt{0+36} = \sqrt{36} = 6$

So, Diagonals of ABCD are also equal. … (2)

From (1) and (2), we can definitely say that ABCD is a square.

Therefore, Champa is correct.

6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer.
(i) (–1, –2), (1, 0), (–1, 2), (–3, 0)
(ii) (–3, 5), (3, 1), (0, 3), (–1, –4)
(iii) (4, 5), (7, 6), (4, 3), (1, 2)
Ans. (i) Let A = (–1, –2), B = (1, 0), C= (–1, 2) and D = (–3, 0)
Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[1-(-1)]^2+[0-(-2)]^2} = \sqrt{(2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

BC = $\sqrt{[-1-1]^2+[2-0]^2} = \sqrt{(-2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

CD = $\sqrt{[-3-(-1)]^2+[0-2]^2} = \sqrt{(-2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

DA = $\sqrt{[-3-(-1)]^2+[0-(-2)]^2} = \sqrt{(-2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Therefore, all four sides of quadrilateral are equal. … (1)

Now, we will check the length of diagonals.

AC = $\sqrt{[-1-(-1)]^2+[2-(-2)]^2} = \sqrt{(0)^2+(4)^2} = \sqrt{0+16} = \sqrt{16} = 4$

BD = $\sqrt{[-3-1]^2+[0-0]^2} = \sqrt{(-4)^2+(0)^2} = \sqrt{16+0} = \sqrt{16} = 4$

Therefore, diagonals of quadrilateral ABCD are also equal. … (2)

From (1) and (2), we can say that ABCD is a square.

(ii) Let A = (–3, 5), B= (3, 1), C= (0, 3) and D= (–1, –4)
Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[3-(-3)]^2+[1-5]^2} = \sqrt{(6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

BC = $\sqrt{[0-3]^2+[3-1]^2} = \sqrt{(-3)^2+(2)^2} = \sqrt{9+4} = \sqrt{13}$

CD = $\sqrt{[-1-0]^2+[-4-3]^2} = \sqrt{(-1)^2+(-7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$

DA = $\sqrt{[-1-(-3)]^2+[-4-5]^2} = \sqrt{(2)^2+(-9)^2} = \sqrt{4+81} = \sqrt{85}$

We cannot find any relation between the lengths of different sides.

Therefore, we cannot give any name to the quadrilateral ABCD.

(iii) Let A = (4, 5), B= (7, 6), C= (4, 3) and D= (1, 2)
Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[7-4]^2+[6-5]^2} = \sqrt{(3)^2+(1)^2} = \sqrt{9+1} = \sqrt{10}$

BC = $\sqrt{[4-7]^2+[3-6]^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

CD = $\sqrt{[1-4]^2+[2-3]^2} = \sqrt{(-3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

DA = $\sqrt{[1-4]^2+[2-5]^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Here opposite sides of quadrilateral ABCD are equal. … (1)

We can now find out the lengths of diagonals.

AC = $\sqrt{[4-4]^2+[3-5]^2} = \sqrt{(0)^2+(-2)^2} = \sqrt{0+4} = \sqrt{4} = 2$

BD = $\sqrt{[1-7]^2+[2-6]^2} = \sqrt{(-6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

Here diagonals of ABCD are not equal. … (2)

From (1) and (2), we can say that ABCD is not a rectangle therefore it is a parallelogram.

7. Find the point on the x–axis which is equidistant from (2, –5) and (–2, 9).
Ans. Let the point be (x, 0) on x–axis which is equidistant from (2, –5) and (–2, 9).
Using Distance Formula and according to given conditions we have:

$$\sqrt{[x-2]^2+[0-(-5)]^2} = \sqrt{[x-(-2)]^2+[(0-9)]^2}$$

$\Rightarrow \sqrt{x^2+4-4x+25} = \sqrt{x^2+4+4x+81}$

Squaring both sides, we get

$\Rightarrow x^2 + 4 - 4x + 25 = x^2 + 4 + 4x + 81$

$\Rightarrow -4x + 29 = 4x + 85$

$\Rightarrow 8x = -56$

$\Rightarrow x = -7$

Therefore, point on the x–axis which is equidistant from (2, –5) and (–2, 9) is (–7, 0)

8. Find the values of y for which the distance between the points P (2, –3) and Q (10, y) is 10 units.
Ans. Using Distance formula, we have
$10 = \sqrt{(2-10)^2+(-3-y)^2}$

$\Rightarrow 10 = \sqrt{(-8)^2+9+y^2+6y}$

$\Rightarrow 10 = \sqrt{64 + 9 + y^2 + 6y}$

Squaring both sides, we get

100 = $73 + y^2 + 6y$

$\Rightarrow y^2 + 6y - 27 = 0$

Solving this Quadratic equation by factorization, we can write

$\Rightarrow y^2 + 9y - 3y - 27 = 0$

⇒ y (y + 9) – 3 (y + 9) = 0

⇒ (y + 9) (y − 3) = 0

⇒ y = 3, −9

9. If, Q (0, 1) is equidistant from P (5, –3) and R (x, 6), find the values of x. Also, find the distances QR and PR.

Ans. It is given that Q is equidistant from P and R. Using Distance Formula, we get
PQ = RQ

$\Rightarrow PQ^2 = RQ^2$

$\Rightarrow \sqrt{(0-5)^2 + [1-(-3)^2]} = \sqrt{(0-x)^2 + (1-6)^2}$

$\Rightarrow \sqrt{(-5)^2 + [4^2]} = \sqrt{(x)^2 + (-5)^2}$

$\Rightarrow \sqrt{25+16} = \sqrt{x^2 + 25}$

Squaring both sides, we get

⇒ 25 + 16 = $x^2 + 25$

$\Rightarrow x^2 = 16$

⇒ x = 4, −4

Thus, Q is (4, 6) or (–4, 6).

Using Distance Formula to find QR, we get

Using value of x = 4 QR = $\sqrt{(4-0)^2 + [6-1^2]} = \sqrt{16+25} = \sqrt{41}$

Using value of x = –4 QR = $\sqrt{(-4-0)^2 + [6-1^2]} = \sqrt{16+25} = \sqrt{41}$

Therefore, QR = $\sqrt{41}$

Using Distance Formula to find PR, we get

Using value of x = 4 PR = $\sqrt{(4-5)^2 + [6-(-3)^2]} = \sqrt{1+81} = \sqrt{82}$

Using value of x =–4 PR = $\sqrt{(-4-5)^2 + [6-(-3)^2]} = \sqrt{81+81} = \sqrt{162} = 9\sqrt{2}$

Therefore, x = 4, –4

QR = $\sqrt{41}$, PR = $\sqrt{82}, 9\sqrt{2}$

10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (–3, 4).

Ans. It is given that (x, y) is equidistant from (3, 6) and (–3, 4).
Using Distance formula, we can write

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{[x-(-3)]^2 + (y-4)^2}$$

$\Rightarrow \sqrt{x^2 + 9 - 6x + y^2 + 36 - 12y} = \sqrt{x^2 + 9 + 6x + y^2 + 16 - 8y}$

Squaring both sides, we get

$\Rightarrow x^2 + 9 - 6x + y^2 + 36 - 12y$

$= x^2 + 9 + 6x + y^2 + 16 - 8y$

$\Rightarrow -6x - 12y + 45$

$= 6x - 8y + 25$

$\Rightarrow 12x + 4y = 20$

$\Rightarrow 3x + y = 5$